



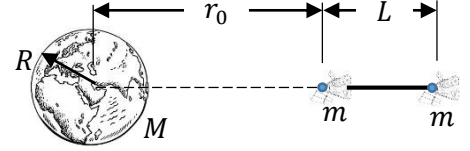
PHYS 101 – General Physics I Midterm Exam

Duration: 120 minutes

Saturday, 23 November 2024; 09:30

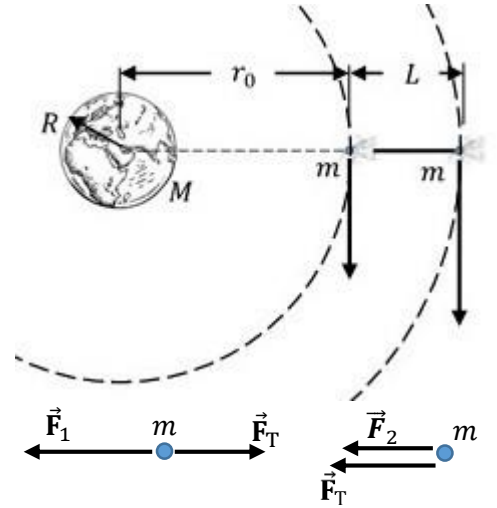
1. Two identical satellites of mass m are tied with an inextensible rope of length L . The mass of the rope is negligible. Assume that the two satellites are near a planet of mass M and radius R . The mass of the planet is so much larger than the satellite mass that the gravitational attraction between the satellites can be neglected in your calculations.

(a) (8 Pts.) Assume that initially both satellites are stationary, and the two satellites and the center of the planet are on a straight line as shown in the figure. The closer satellite is at distance r_0 from the center of the planet, and the other satellite is at $r_0 + L$, as shown in the figure. The satellites are released from this configuration, accelerate towards the planet and crash to the surface. What is the tension in the rope as a function of r (the distance of the closer satellite to the center of the planet) after the satellites are released?



(b) (8 Pts.) What is the speed of the satellites when the lower satellite crashes to the surface?

(c) (9 Pts.) Now assume that instead of being released with zero velocity, satellites are given initial velocities so that they are in circular orbits about the planet. Both satellites go through uniform circular motion so that the center of the planet and the two satellites always remain on a straight line, and the closer satellite is always at distance r_0 from the center of the planet. What is the period of this orbit?



Solution:

(a) When the closer satellite is at a distance r from the center of the planet, magnitudes of the gravitational force acting on the satellites are

$$F_1 = G \frac{Mm}{r^2}, \quad F_2 = G \frac{Mm}{(r+L)^2}.$$

Since the satellites are connected by an inextensible rope, their accelerations are equal. Considering the free-body diagrams for the two satellites, we have

$$F_1 - F_T = ma, \quad F_2 + F_T = ma \rightarrow F_T = \frac{1}{2}(F_1 - F_2) = \frac{GMm}{2} \left(\frac{1}{r^2} - \frac{1}{(r+L)^2} \right) = \frac{GMm}{2} \left(\frac{L^2 + 2Lr}{r^2(r+L)^2} \right).$$

(b) Mechanical energy is conserved during the fall. Satellites start from rest. Therefore, initial energy is

$$E_i = U_i = -GMm \left(\frac{1}{r_0} + \frac{1}{r_0 + L} \right).$$

At the instant the closer satellite strikes the surface of the planet

$$E_f = K_f + U_f = \frac{1}{2}(2m)v^2 - GMm \left(\frac{1}{R} + \frac{1}{R+L} \right).$$

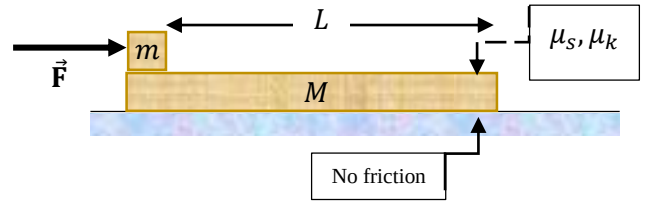
$$E_f = E_i \rightarrow v^2 = GM \left(\frac{1}{R} + \frac{1}{R+L} \right) - GM \left(\frac{1}{r_0} + \frac{1}{r_0 + L} \right) \rightarrow v = \sqrt{GM(r_0 - R)} \sqrt{\frac{1}{Rr_0} + \frac{1}{(R+L)(r_0 + L)}}.$$

(c) Both satellites are in circular orbits and have the same period. This means,

$$G \frac{Mm}{r_0^2} - F_T = mr_0\omega^2, \quad G \frac{Mm}{(r_0 + L)^2} + F_T = m(r_0 + L)\omega^2 \rightarrow \omega = \sqrt{\left(\frac{GM}{2r_0 + L} \right) \left(\frac{2r_0^2 + 2r_0L + L^2}{r_0^2(r_0 + L)^2} \right)}.$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi r_0(r_0 + L)}{\sqrt{GM}} \sqrt{\frac{2r_0 + L}{2r_0^2 + 2r_0L + L^2}}.$$

2. As illustrated in the figure, a smaller block with mass m rests atop a larger block with mass M . The larger block is free to slide on a perfectly smooth, frictionless horizontal surface. Coefficients of static and kinetic friction between the two blocks are denoted as μ_s and μ_k , respectively. Initially, both blocks are at rest. At time $t = 0$, a constant horizontal force begins acting on the upper block. The gravitational acceleration is given as g .



(a) (5 Pts.) Draw free-body diagrams for both blocks.

(b) (5 Pts.) Determine the maximum value of the magnitude of $\vec{F}$ for which that the top block remains at rest relative to the bottom block.

(c) (5 Pts.) Assume that the force $\vec{F}$ is large enough and the top block starts to slide on the bottom one. What are the accelerations of the two blocks?

(d) (5 Pts) Find the work done by the force $\vec{F}$ on the sliding top block until it reaches the other end of the bottom block in terms of the accelerations a_1, a_2 , the force F and L ?

(e) (5 Pts.) What is the average power lost to heat until the top block reaches the other end of the bottom block in terms of the quantities a_1, a_2, M and L ?

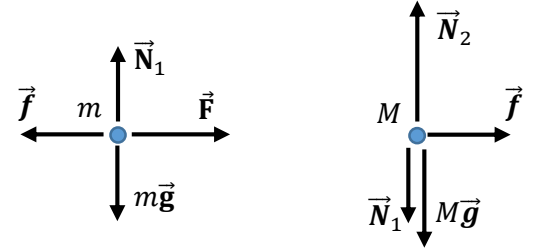
Solution:

(a)

(b) Since there is no relative motion, accelerations of the two blocks are equal.

$$F - f = ma, \quad f = Ma \rightarrow a = \frac{F}{M + m}, \quad f = \frac{MF}{M + m}$$

$$f \leq \mu_s N_1 \rightarrow f \leq \mu_s mg \rightarrow \frac{MF}{M + m} \leq \mu_s mg \rightarrow F \leq \left(\frac{M + m}{M} \right) \mu_s mg$$



(c) In this case the top block slides on the bottom one. Therefore, accelerations of the two block are different.

$$F - f = ma_1, \quad f = Ma_2, \quad f = \mu_k mg \rightarrow a_1 = \frac{F - \mu_k mg}{m}, \quad a_2 = \frac{m}{M} \mu_k g$$

(d) Relative acceleration of the top block with respect to the bottom one is $a_1 - a_2$. The top block covers the distance L in time T , where

$$\frac{1}{2}(a_1 - a_2)T^2 = L \rightarrow T = \sqrt{\frac{2L}{a_1 - a_2}}.$$

During this time the top block is displaced by $d = a_1 T^2 / 2 = a_1 L / (a_1 - a_2)$ with respect to the floor, and the work done by the force on the top block is

$$W_F = Fd = \frac{a_1 FL}{a_1 - a_2}.$$

(e) Friction force does positive work on the bottom block while it does negative work on the top block. Energy dissipated as heat is equal to the work done on the top block by the friction force as the top block covers the distance L relative to the bottom block.

$$W_{f_{top}} = -fL = -Ma_2 L$$

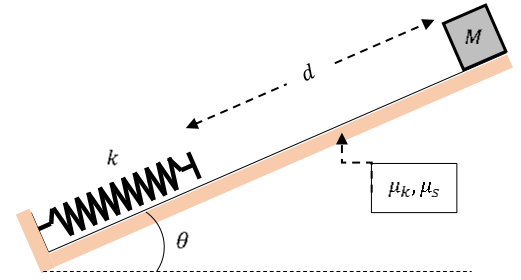
$$\bar{P}_H = \frac{|W_{f_{top}}|}{T} \rightarrow \bar{P}_H = fL \sqrt{\frac{a_1 - a_2}{2L}} = Ma_2 \sqrt{\frac{L}{2}(a_1 - a_2)}.$$

3. A box of mass M starts from rest and slides a distance d down an inclined plane with angle of inclination θ before colliding with a spring. The coefficient of kinetic friction between the box and the surface is μ_k , such that $\tan \theta > \mu_s \geq \mu_k$.

(a) (8 Pts.) What is the speed of the box just before it hits the spring?

(b) (9 Pts.) If the box compresses the spring a distance X from its equilibrium position before coming to rest, what is the stiffness constant of the spring?

(c) (8 Pts.) We observe that after it stops momentarily, the compressed spring can push the block back up the inclined plane. What is the maximum value of the coefficient of static friction μ_s between the box and the surface, so that this can happen?



Solution:

(a) If $\tan \theta > \mu_s \geq \mu_k$, the box will start from rest and slide down the inclined plane. Because of friction, mechanical energy is not conserved. Choosing the zero level of the gravitational potential energy at the equilibrium position of the top of the spring, we have

$$\Delta E = W_f \rightarrow \frac{1}{2} M v^2 - M g d \sin \theta = -\mu_k M g \cos \theta d \rightarrow v = \sqrt{2 g d (\sin \theta - \mu_k \cos \theta)}$$

(b) Using the work-energy relation $\Delta E = W_f$ once more, we have

$$\frac{1}{2} k X^2 - M g X \sin \theta - \frac{1}{2} M v^2 = -\mu_k M g X \cos \theta \rightarrow k = \frac{2}{X^2} \left(\frac{1}{2} M v^2 + M g X \sin \theta - \mu_k M g X \cos \theta \right)$$

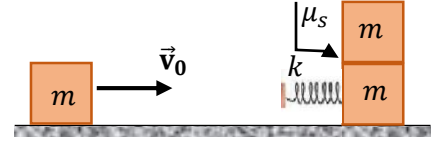
$$k = \frac{2 M g (X + d) (\sin \theta - \mu_k \cos \theta)}{X^2}$$

(c) The compressed spring can push the block back up the inclined plane if the force of the spring satisfies

$$k X \geq M g \sin \theta + \mu_s M g \cos \theta \rightarrow \mu_s \geq \frac{k X}{M g \cos \theta} - \tan \theta .$$

4. Two identical blocks of mass m are on a frictionless horizontal table. Another block of the same mass m is on top of one of the blocks, which also has an ideal spring of spring constant k attached, as shown in the figure. The static friction coefficient between the top and the bottom blocks is μ_s . The blocks stacked on top of each other remain stationary while the other block is moving towards them with speed v_0 . Assume that the top block does not slide on the bottom block.

- (a) (8 Pts.) What are the final velocities of masses in the system?
 (b) (8 Pts.) What is the maximum compression of the spring during the collision?
 (c) (9 Pts.) What is the maximum speed v_0 so that the top block does not slide on the bottom block during the collision?



Solution:

(a) Because of the spring the collision is elastic. Therefore, both linear momentum and kinetic energy is conserved in the collision.

$$mv_0 = mv'_1 + 2mv'_2, \quad \frac{1}{2}mv_0^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}(2m)v_2'^2 \text{ or } v_0 = -v'_1 + v'_2$$

$$v'_1 = -\frac{1}{3}v_0, \quad v'_2 = \frac{2}{3}v_0.$$

(b) Maximum compression occurs when the relative velocity of the colliding objects is zero, or when they have the same velocity.

$$mv_0 = (m + 2m)v' \rightarrow v' = \frac{1}{3}v_0$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m + 2m)v'^2 + \frac{1}{2}k\Delta^2 \rightarrow \Delta = v_0\sqrt{\frac{2m}{3k}}$$

(c) The force which keeps the top block from sliding is the static friction force, which has an upper bound. $f_s \leq \mu_s mg$. Therefore, for the top block $ma \leq \mu_s mg \rightarrow a \leq \mu_s g$, which gives an upper bound for the acceleration of the top block without sliding. When the spring starts to compress, it will cause the two blocks on top of each other to accelerate. Maximum acceleration occurs when the force is maximum at maximum compression, found in part (b). This force will accelerate both blocks, so

$$k\Delta = 2ma, \quad kv_0\sqrt{\frac{2m}{3k}} = 2ma \rightarrow a = \sqrt{\frac{k}{6m}}v_0.$$

Using the result $a \leq \mu_s g$, we get

$$\sqrt{\frac{k}{6m}}v_0 \leq \mu_s g \rightarrow v_0 \leq \sqrt{\frac{6m}{k}}\mu_s g \rightarrow v_{0\max} = \sqrt{\frac{6m}{k}}\mu_s g.$$